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Matrix Elements of Electroweak Penguin Operators in the $1/N_c$ Expansion

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Abstract

It is shown that the $K \to \pi\pi$ matrix elements of the four–quark operator Q_7, generated by the electroweak penguin–like diagrams of the Standard Model, can be calculated to first non–trivial order in the chiral expansion and in the $1/N_c$ expansion. Although the resulting B factors $B_7^{(1/2)}$ and $B_7^{(3/2)}$ are found to depend only logarithmically on the matching scale μ, their actual numerical values turn out to be rather sensitive to the precise choice of μ in the GeV region. We compare our results to recent numerical evaluations from lattice–QCD and to other model estimates.

1 Introduction

In the Standard Model, the physics of non–leptonic K–decays is described by an effective Lagrangian which is the sum of four–quark operators Q_i modulated by c–number coefficients c_i (Wilson coefficients)

$$\mathcal{L}_{\rm eff}^{|\Delta S|=1} = -\frac{G_{\rm F}}{\sqrt{2}} V_{ud} V_{us}^* \sum_i c_i(\mu) Q_i(\mu) + \text{h.c.}\,, \tag{1}$$

where $G_{\rm F}$ is the Fermi constant and V_{ud}, V_{us} are the appropriate matrix elements of flavour mixing. This is the effective Lagrangian which results after integrating out the fields in the Standard Model with heavy masses ($Z^0, W^\pm, t, b$ and c), in the presence of the strong interactions evaluated in perturbative QCD (pQCD) down to a scale μ below the mass of the charm quark M_c. The scale μ has to be large enough for the pQCD evaluation of the coefficients c_i to be valid and, therefore, it is much larger than the scale at which an effective Lagrangian description in terms of the Nambu–Goldstone degrees of freedom (K, π and η) of the spontaneous $SU(3)_L \times SU(3)_R$ symmetry breaking (SχSB) is appropriate. Furthermore, the evaluation of the coupling constants of the low–energy effective chiral Lagrangian which describes strangeness changing $|\Delta S| = 1$ transitions cannot be made within pQCD because at scales $\mu \lesssim 1\,\text{GeV}$ we enter a regime where SχSB and confinement take place and the dynamics of QCD is then fully governed by non–perturbative phenomena.

In this letter we shall be concerned with two of the four–quark operators in the Lagrangian in eq. (1), the operators

$$Q_7 = 6(\bar{s}_L \gamma^\mu d_L) \sum_{q=u,d,s} e_q (\bar{q}_R \gamma_\mu q_R)\,, \tag{2}$$

and

$$Q_8 = -12 \sum_{q=u,d,s} e_q (\bar{s}_L q_R)(\bar{q}_R d_L)\,, \tag{3}$$

where e_q denote quark charges in units of the electric charge, and summation over quark colour indices within brackets is understood. The operator Q_7 emerges at the M_W scale from considering the so–called electroweak penguin diagrams. In the presence of the strong interactions, the renormalization group evolution of Q_7 from the scale M_W down to a scale $\mu \lesssim M_c$ mixes, in particular, this operator with the four–quark density–density operator Q_8. These two operators, times their corresponding Wilson coefficients, contribute to the lowest order $\mathcal{O}(p^0)$ effective chiral Lagrangian which induces $|\Delta S| = 1$ transitions in the presence of electromagnetic interactions to order $\mathcal{O}(\alpha)$ and of virtual Z^0 exchange, i.e., the Lagrangian [1]

$$\mathcal{L}_{\chi,0}^{|\Delta S|=1} = -\frac{G_{\rm F}}{\sqrt{2}} \frac{\alpha}{\pi} V_{ud} V_{us}^* \frac{M_\rho^6}{16\pi^2}\, \mathbf{h}\, \text{tr}\left(U \lambda_L^{(23)} U^\dagger Q_R\right) + \text{h.c.}\,. \tag{4}$$

Here, U is the matrix field which collects the octet of pseudoscalar Goldstone fields, $Q_R = \text{diag}[2/3, -1/3, -1/3]$ is the right–handed charge matrix associated with the electromagnetic couplings of the light quarks, and $\lambda_L^{(23)}$ is the effective left–handed flavour matrix $\left(\lambda_L^{(23)}\right)_{ij} = \delta_{i2}\delta_{3j}$ $(i,j = 1,2,3)$. Under chiral rotations (V_L, V_R):

$$U \to V_R U V_L^\dagger\,, \quad Q_R \to V_R Q_R V_R^\dagger\,, \quad \lambda_L^{(23)} \to V_L \lambda_L^{(23)} V_L^\dagger\,, \tag{5}$$

and the trace on the r.h.s. of eq. (4) is an invariant. Actually, this is the only possible invariant which in the Standard Model can generate $|\Delta S| = 1$ transitions to orders $\mathcal{O}(\alpha)$ and $\mathcal{O}(p^0)$ in

the chiral expansion. With the given choice of the overall normalization factor in front of the r.h.s. of eq. (4), the coupling constant **h** is dimensionless and, *a priori*, of order $\mathcal{O}(N_c^2)$ in the $1/N_c$ expansion [2–4]. This coupling constant plays a crucial rôle in the phenomenological analysis of radiative corrections to the $K \to \pi\pi$ amplitudes [5]. The determination of **h** is needed for a reliable estimate of these corrections [1]. The purpose of this note is to show that, following recent work reported in ref. [7], one can calculate the $K \to \pi\pi$ matrix elements of the Q_7 operator (and, therefore, the so called $B_7^{(1/2)}$ and $B_7^{(3/2)}$ factors) to first non–trivial order in the chiral expansion and the $1/N_c$ expansion [2]. This implies that (at least) the contribution to the constant **h** from the Q_7 and Q_8 terms in the four–quark effective Lagrangian can be calculated to first non–trivial order in the $1/N_c$ expansion, a first step towards the required goal. The leading $\mathcal{O}(N_c^2)$ contribution to **h** vanishes trivially. It could only come from the bosonization of the factorized Q_8 operator times its Wilson coefficient, but this coefficient is subleading at large N_c [3]. The next–to–leading contribution comes from the bosonization of the unfactorized Q_7 operator and its mixing via gluonic interactions with the Q_8 operator. It involves the same two–point function which governs the electroweak $\pi^+ - \pi^0$ mass difference in large–N_c QCD and this is why we are able to compute the contributions from the Q_7 and Q_8 terms of the four–quark Lagrangian to the constant **h** at the stated order in the $1/N_c$ expansion. The bosonization of the factorized Q_7 operator and of the unfactorized Q_8 operator can only contribute to terms of order $\mathcal{O}(p^2)$ (or higher) in the chiral expansion and they are, therefore, inoperational in the calculation of **h**. It turns out, however, that there is only a partial cancellation of the μ–dependences generated by the product of the bosonization of the unfactorized Q_7 operator times its Wilson coefficient c_7 with the μ–dependence coming from the product of the Wilson coefficient c_8, which is non–leading in the $1/N_c$ expansion, times the bosonization of the factorized Q_8 operator. The full cancellation of μ–dependences requires the consideration, as well, of the bosonization of other four–quark operators (in particular the unfactorized Q_2 operator) in the presence of the electroweak interactions. This involves integrals of four–point functions which we have not yet fully explored within the framework of the $1/N_c$ expansion. The constant **h** gets, therefore, contributions from other operators than just Q_7 and Q_8, and which we have not computed so far. We shall, therefore, concentrate here on the calculation of the factors $B_7^{(1/2)}$ and $B_7^{(3/2)}$, and on their comparison to recent lattice QCD determinations [10–12], as well as to recent analytic determinations which have been made [4] using the "effective action approach" of ref. [14].

2 Bosonization of Q_7 and Q_8

As already mentioned, the bosonization of the operator Q_7 is needed to next–to–leading order in the $1/N_c$ expansion. The problem is entirely analogous to the bosonization of the operator $Q_{LR} \equiv (\bar{q}_L \gamma^\mu Q_L q_L)\,(\bar{q}_R \gamma^\mu Q_R q_R)$ which governs the electroweak $\pi^+ - \pi^0$ mass difference and which has been recently discussed in ref. [7]. Because of the LR structure, the factorized component of the operator Q_7, which is leading in $1/N_c$, cannot contribute to order $\mathcal{O}(p^0)$ in the low–energy effective Lagrangian. The first non–trivial contribution from this operator is

[1] See ref. [6] for a recent discussion of the size of these corrections.

[2] A similar observation has also been made by J. Donoghue [8].

[3] See e.g. Buras's lectures [9].

[4] See e.g. ref. [13] and references therein.

next–to–leading in the $1/N_c$ expansion and is given by the integral [5]

$$Q_7 \to -3ig_{\mu\nu}\int \frac{d^4q}{(2\pi)^4}\Pi_{LR}^{\mu\nu}(q)\ \mathrm{tr}\left(U\lambda_L^{(23)}U^\dagger Q_R\right)\,, \tag{6}$$

involving the two–point function

$$\Pi_{LR}^{\mu\nu}(q) = 2i\int d^4x\, e^{iq\cdot x}\langle 0|\mathrm{T}\left(L^\mu(x)R^\nu(0)^\dagger\right)|0\rangle\,, \tag{7}$$

with currents

$$L^\mu = \bar{q}_i(x)\gamma^\mu\frac{1}{2}(1-\gamma_5)q_j(x) \quad \text{and} \quad R^\mu = \bar{q}_i(x)\gamma^\mu\frac{1}{2}(1+\gamma_5)q_j(x)\,, \tag{8}$$

and i and j fixed flavour indices, $i \neq j$. This integral, which has to be evaluated in the chiral limit, where ($Q^2 = -q^2$)

$$\Pi_{LR}^{\mu\nu}(q) = (q^\mu q^\nu - g^{\mu\nu}q^2)\Pi_{LR}(Q^2)\,, \tag{9}$$

is divergent for large Q^2 and needs to be regulated. Before discussing this point, let us recall that in the large–N_c limit, the spectral function associated with $\Pi_{LR}(Q^2)$ consists of the difference of an infinite number of narrow vector states and an infinite number of narrow axial–vector states, together with the Goldstone pion pole:

$$\frac{1}{\pi}\mathrm{Im}\Pi_{LR}(t) = \sum_V f_V^2 M_V^2\delta(t-M_V^2) - \sum_A f_A^2 M_A^2\delta(t-M_A^2) - f_\pi^2\delta(t)\,. \tag{10}$$

Since $\Pi_{LR}(Q^2)$ obeys an unsubtracted dispersion relation, we find that

$$-Q^2\Pi_{LR}(Q^2) = f_\pi^2 + \sum_A f_A^2 M_A^2\frac{Q^2}{M_A^2+Q^2} - \sum_V f_V^2 M_V^2\frac{Q^2}{M_V^2+Q^2}\,. \tag{11}$$

Furthermore, in the chiral limit of QCD, the operator product expansion (OPE) applied to the correlation function $\Pi_{LR}(Q^2)$ implies

$$\lim_{Q^2\to\infty} Q^2\Pi_{LR}(Q^2) \to 0 \quad \text{and} \quad \lim_{Q^2\to\infty} Q^4\Pi_{LR}(Q^2) \to 0\,. \tag{12}$$

These relations result in the two Weinberg sum rules [15]

$$\sum_V f_V^2 M_V^2 - \sum_A f_A^2 M_A^2 = f_\pi^2 \quad \text{and} \quad \sum_V f_V^2 M_V^4 - \sum_A f_A^2 M_A^4 = 0\,. \tag{13}$$

The usual prescription [16] for the evaluation of integrals such as (6) consists in taking a sharp cut-off in the (euclidian) integration over Q^2,

$$Q_7 \to -6\frac{3}{32\pi^2}\int_0^{\Lambda^2} dQ^2Q^2\left(-Q^2\Pi_{LR}(Q^2)\right)\mathrm{tr}\left(U\lambda_L^{(23)}U^\dagger Q_R\right)\,. \tag{14}$$

[5] Up to the replacement of Q_L by $\lambda_L^{(23)}$, the bosonization of the operator Q_7 follows from the same procedure as described for the operator Q_{LR} in ref. [7].

With the constraints between the couplings and masses of the narrow states coming from the Weinberg sum rules (13), the integral on the r.h.s. of eq. (6) becomes then only logarithmically dependent on the ultraviolet scale Λ, with the following result

$$\int_0^{\Lambda^2} dQ^2 Q^2 \left(-Q^2 \Pi_{LR}(Q^2)\right) = \left[\sum_A f_A^2 M_A^6 \log\frac{\Lambda^2}{M_A^2} - \sum_V f_V^2 M_V^6 \log\frac{\Lambda^2}{M_V^2}\right] . \qquad (15)$$

Notice that if only the contribution from the Goldstone pole had been taken into account, the resulting expression would have displayed a polynomial dependence on the cut-off scale Λ. Another possibility is to evaluate the integral (6) within a dimensional regularization scheme, say $\overline{MS}$, in which case one obtains the same result (15), but with the correspondence between the cut–off Λ and the $\overline{MS}$ subtraction scale μ given by

$$\Lambda = \mu \cdot e^{\frac{1}{6}} . \qquad (16)$$

On the other hand, at the level of approximation that we want to attain, the bosonization of the Q_8 operator is only required to leading order in the $1/N_c$ expansion, because its Wilson coefficient is already subleading. To that order and to order $\mathcal{O}(p^0)$ in the chiral expansion it can be readily obtained from the bosonization of the factorized density currents, with the result [6]

$$Q_8 \to -12\left(2B\frac{f_\pi^2}{4}\right)^2 \mathrm{tr}\left(U\lambda_L^{(23)}U^\dagger Q_R\right) , \qquad (17)$$

where B is the low energy constant which describes the bilinear single flavour quark condensate in the chiral limit, $B = -\langle\bar\psi\psi\rangle/f_\pi^2$.

As discussed in ref. [18] there is a constraint that emerges in the large–N_c limit which relates the leading $d = 6$ order parameter in the OPE of the Π_{LR} two–point function [19] to couplings and masses of the narrow states:

$$\begin{aligned}\lim_{Q^2\to\infty} Q^6\Pi_{LR}(Q^2) &= -4\pi^2\left(\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right)\langle\bar\psi\psi\rangle^2 \\ &= \sum_V f_V^2 M_V^6 - \sum_A f_A^2 M_A^6 . \end{aligned} \qquad (18)$$

This relation provides part of the cancellation between the μ dependence of the bosonization of the operator $Q_7(\mu)$ with the short–distance dependence on μ in the Wilson coefficient of Q_8; but, as already mentioned, and contrary to the simple case of the electroweak contribution to the $\pi^+ - \pi^0$ mass difference discussed in ref. [7], this cancellation here is incomplete. In this respect, we wish to comment on an important point concerning the scale dependence in the relation in eq. (18). The term on the r.h.s. of the first line results from a lowest order pQCD calculation of the Wilson coefficient. Using the renormalization group improvement to one loop, this result becomes

$$-4\pi^2\frac{\alpha_s}{\pi}\langle\bar\psi\psi\rangle^2 \to -4\pi^2\langle\widehat{\bar\psi\psi}\rangle^2\frac{1}{-\beta_1}\left(\frac{-\beta_1\alpha_s(Q^2)}{\pi}\right)^{\frac{2\gamma_1+\beta_1}{\beta_1}} , \qquad (19)$$

with $\langle\widehat{\bar\psi\psi}\rangle$ the scale invariant quark condensate (the analog to invariant quark masses). In the large–N_c limit, $\beta_1 \to \frac{-11}{6}N_c$ and $\gamma_1 \to \frac{3}{4}N_c$. The Q^2 dependence of the one–loop result

[6] See e.g. the lectures in ref. [17] and references therein.

is indeed rather mild $\sim \left(\log Q^2/\Lambda^2_{\overline{\rm MS}}\right)^{-2/11}$, but it does not go to a constant as the exact large–N_c result in the second line of eq. (18) demands. This mismatch is due to the fact that pQCD is at best an approximation. It may happen that a two–loop renormalization group improvement of the OPE result approaches a constant behaviour at large Q^2 in a better way. In any case, this is a typical example of unavoidable mismatches that one will encounter between non–perturbative evaluations of matrix elements and short–distance pQCD evaluations, which are necessarily only approximate. In general, however, it is well known that at higher orders ambiguities will mix the short–distance coefficients of different powers in the OPE. It is difficult to imagine how to avoid these uncertainties in a final matching between short–distances and long–distances unless a breakthrough is made in understanding the relationship between pQCD and full QCD.

Finally, if we restrict ourselves to the Lowest Meson Dominance (LMD) approximation to large-N_c QCD discussed in ref. [20] and identify M_V with M_ρ, our calculation (in the $\overline{MS}$ scheme) gives the following contributions to the constant $\mathbf{h}$ coming from the operators Q_7 and Q_8,

$$\frac{\alpha}{\pi}\mathbf{h}[Q_7] = -18c_7(\mu)\frac{f_\pi^2}{M_\rho^2}\left[\log\frac{\Lambda^2}{M_\rho^2} - 2\log 2\right] \quad \text{and} \quad \frac{\alpha}{\pi}\mathbf{h}[Q_8] = -48\pi^2 c_8(\mu)\frac{\langle\bar\psi\psi\rangle^2(\mu)}{f_\pi^2 M_\rho^6}\,, \quad (20)$$

where the relation (16) is understood.

3 The B Factors $B_7^{(1/2)}$ and $B_7^{(3/2)}$

The bosonic expression of Q_7 given by eqs. (14) and (15) enables us to compute the $K\to\pi\pi$ matrix elements induced by this operator which, following the usual conventions, we express in terms of the following isospin amplitudes

$$\langle Q_7\rangle_I \equiv \langle(\pi\pi)_I|Q_7|K^0\rangle\,, \qquad I = 0, 2\,. \quad (21)$$

To leading order $\mathcal{O}(p^0)$ in the chiral expansion and to next–to–leading order in the $1/N_c$ expansion, $\mathcal{O}(1/\sqrt{N_c})$ for $K\to\pi\pi$ amplitudes, we obtain the following result

$$\langle Q_7\rangle_0 = \sqrt{2}\langle Q_7\rangle_2 = \frac{6\sqrt{3}}{16\pi^2 f_\pi^3}\left[\sum_A f_A^2 M_A^6\log\frac{\Lambda^2}{M_A^2} - \sum_V f_V^2 M_V^6\log\frac{\Lambda^2}{M_V^2}\right]. \quad (22)$$

It has become customary (rather unfortunately) to parameterize the results of weak matrix elements of four–quark operators Q_i in terms of the factorized contributions from the so–called vacuum saturation approximation (VSA), modulated by correction factors $B_i^{(\Delta I)}$, $\Delta I = 1/2, 3/2$. In the case of Q_7 and Q_8, one then has

$$\langle Q_7\rangle_0^{\rm VSA} = \frac{1}{2}X + \frac{1}{2N_c}(Z+4Y)\,, \quad \langle Q_7\rangle_2^{\rm VSA} = -\frac{\sqrt{2}}{2}X + \frac{\sqrt{2}}{N_c}Y\,, \quad (23)$$

and

$$\langle Q_8\rangle_0^{\rm VSA} = \frac{1}{2}(Z+4Y) + \frac{1}{2N_c}X\,, \ \langle Q_8\rangle_2^{\rm VSA} = \sqrt{2}Y - \frac{\sqrt{2}}{2N_c}X\,. \quad (24)$$

The quantities X, Y and Z which appear in the above expressions are the same as those usually found in the literature, i.e.,

$$\begin{aligned} X &= \sqrt{3} f_\pi (M_K^2 - M_\pi^2) + \mathcal{O}(p^4)\,, \\ Y &= \sqrt{3} f_\pi \left(\frac{M_K^2}{m_s + m_d}\right)^2 + \mathcal{O}(p^2)\,, \\ Z &= 4\left(\frac{F_K}{F_\pi} - 1\right) Y + \mathcal{O}(p^4)\,. \end{aligned} \tag{25}$$

There is no theoretical justification to consider the VSA as a good limit of any kind in QCD [7]. This is reflected by the fact that e.g., the terms proportional to $\frac{1}{N_c}$ in eqs. (23) and (24) do not correspond to the correct $1/N_c$ expansion and, in fact, the contributions of Y and Z to $\langle Q_7 \rangle_I^{\rm VSA}$, although suppressed by a factor $1/N_c$, are actually numerically dominant, since (throughout, M_K denotes the neutral kaon mass)

$$Y \,=\, \frac{X}{M_K^2 - M_\pi^2} \left(\frac{M_K^2}{m_s + m_d}\right)^2 \,\sim\, 11.4\, X \,\left(\frac{0.158\,{\rm GeV}}{m_s + m_d}\right)^2 . \tag{26}$$

Furthermore, whereas X is scale independent, Y and Z depend on the $\overline{MS}$ subtraction scale μ through the quark mass term $m_s + m_d$ in the denominator. We shall nevertheless follow these conventions, if only to be able to compare our results to those existing in the literature. The corresponding B factors are then defined as

$$B_i^{(1/2)} \,=\, \frac{\langle Q_i \rangle_0}{\langle Q_i \rangle_0^{\rm VSA}}\,, \quad B_i^{(3/2)} \,=\, \frac{\langle Q_i \rangle_2}{\langle Q_i \rangle_2^{\rm VSA}}\,. \tag{27}$$

In the sequel, we quote our results in the $\overline{MS}$ scheme.

Considering first the operator Q_7, and restricting the sums in eq. (22) to the LMD approximation discussed in ref. [20], the previous calculation leads to the results

$$B_7^{(1/2)}(\mu) \,=\, \frac{X}{X + \frac{1}{N_c}(Z + 4Y)} \left\{ 1 \,+\, \frac{3}{2\pi^2 f_\pi^2} \frac{M_V^4}{M_K^2 - M_\pi^2} \left[\log \frac{\Lambda^2}{M_V^2} - 2 \log 2 \right] \right\}, \tag{28}$$

and

$$B_7^{(3/2)}(\mu) \,=\, \frac{X}{X - \frac{2}{N_c} Y} \left\{ 1 \,-\, \frac{3}{4\pi^2 f_\pi^2} \frac{M_V^4}{M_K^2 - M_\pi^2} \left[\log \frac{\Lambda^2}{M_V^2} - 2 \log 2 \right] \right\}. \tag{29}$$

If in these expressions we take the value $M_V = M_\rho$, we obtain, at the scale $\mu = 2$ GeV usually adopted in lattice calculations,

$$B_7^{(1/2)}(\mu = 2\,{\rm GeV}) \,\sim\, \frac{1}{19.5}(1 \,+\, 23)\,, \quad B_7^{(3/2)}(\mu = 2\,{\rm GeV}) \,\sim\, -\frac{1}{6.6}(1 \,-\, 11.5)\,, \tag{30}$$

for, say, $(m_s + m_d)(\mu = 2\,{\rm GeV}) = 158\,{\rm MeV}$, the conventional reference normalization used in eq. (26). These values are both positive and greater than unity. One should however notice that these numbers are rather sensitive to the choice of the scale μ and/or to the value assigned to M_V : for instance, at $\mu = 2M_V/e^{\frac{1}{6}} \sim 1.3$ GeV the contributions between square brackets

[7] Obviously, it would have been much more reasonable to normalize results to the large–N_c result in the lowest order of the chiral expansion.

on the r.h.s. of eqs. (28) and (29) vanish exactly. In order to illustrate these uncertainties, we show in Fig. 1 the variation of these B factors for a reasonable range of values of the scale μ. The area between the solid lines corresponds to the choice $(m_s+m_d)(\mu = 2\,\mathrm{GeV}) = 158\,\mathrm{MeV}$, with $\Lambda_{\overline{MS}}$ varied between 300 MeV and 450 MeV. The area between the dashed lines reflects the same variation of $\Lambda_{\overline{MS}}$, but for the extreme low value $(m_s + m_d)(\mu = 2\,\mathrm{GeV}) = 100\,\mathrm{MeV}$ quoted in some lattice results [21]. Also shown is a magnification of the region corresponding to values of μ around 2 GeV, the reference scale at which the lattice results are usually given. We find that below $\mu \lesssim 1.3\,\mathrm{GeV}$ the $B_7^{(1/2)}$ and $B_7^{(3/2)}$ factors can even become negative, a result which disagrees, drastically, with the positive values quoted at a "matching scale" of $0.8\,\mathrm{GeV}$ which are found in the constituent chiral quark model [13]. It is, however, not clear how this "matching scale" is related to the $\overline{MS}$ scale, μ, in QCD.

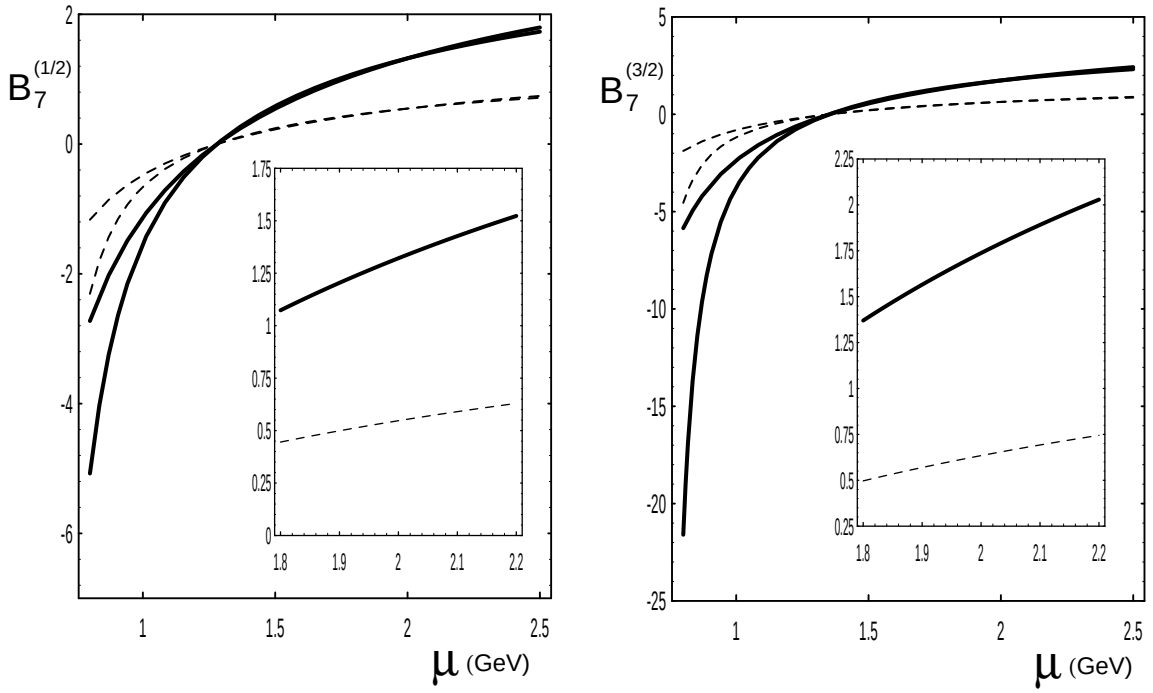

Fig. 1 *The B factors, $B_7^{(1/2)}$ and $B_7^{(3/2)}$, as a function of the μ scale in* GeV. *The solid lines correspond to $(m_s + m_d)(\mu = 2\,\mathrm{GeV}) = 158\,\mathrm{MeV}$; the dashed lines to the extreme low value $(m_s + m_d)(\mu = 2\,\mathrm{GeV}) = 100\,\mathrm{MeV}$.*

Several matrix elements of four–quark operators have also been obtained in numerical simulations of lattice–QCD. These numerical evaluations, however, are based on a yet different definition of the B parameters. The lattice definition uses a current algebra relation between the $K \to \pi\pi$ and the $K \to \pi$ matrix elements which is in fact only valid at order $\mathcal{O}(p^0)$ in the chiral expansion. Thus, in the case of Q_7 discussed here, the lattice–QCD B factor, which we shall distinguish with the symbol tilde on top, is defined by the ratio

$$\widetilde{B}_7^{(3/2)} \equiv \frac{\langle \pi^+ | Q_7^{(3/2)} | K^+ \rangle}{\langle \pi^+ | Q_7^{(3/2)} | K^+ \rangle_0^{\mathrm{VSA}}}\,, \tag{31}$$

where the matrix element in the denominator is evaluated in the chiral limit, as indicated by the subscript "0", and the operator Q_7 has been decomposed into its $\Delta I = 1/2$ and $\Delta I = 3/2$

components, $Q_7 = Q_7^{(1/2)} + Q_7^{(3/2)}$, with

$$Q_7^{(3/2)} \;=\; 2(\bar{s}_L\gamma^\mu d_L)[\bar{u}_R\gamma_\mu u_R - \bar{d}_R\gamma_\mu d_R] \;+\; 2(\bar{s}_L\gamma^\mu u_L)(\bar{u}_R\gamma_\mu d_R)\,. \tag{32}$$

The latest results obtained by various groups are consistent with each other:

$$\widetilde{B}_7^{(3/2)}(\mu = 2\,\mathrm{GeV})\Big|_{\mathrm{latt}} \;=\; \left\{ \begin{array}{l} 0.58^{+5+2}_{-4-8} \\ 0.61(11) \end{array} \right. \tag{33}$$

The first value has been taken from ref. [10], while the second one arises from the results of refs. [11] and [12], translated into the $\overline{MS}$ scheme [22]. In the vacuum saturation approximation which the lattice community uses, one obtains

$$\langle\pi^+|Q_7^{(3/2)}|K^+\rangle_0^{\mathrm{VSA}} \;=\; -\frac{2}{N_c}\frac{\langle\bar{\psi}\psi\rangle^2}{f_\pi^2} \;=\; -\frac{2}{N_c}\frac{f_\pi}{\sqrt{3}}Y\;, \tag{34}$$

where in the second expression, following common practice, we have traded the dependence with respect to the condensate for the dependence on the strange quark mass, using the Gell-Mann–Oakes–Renner relation, which holds in the chiral limit. The calculation based on the bosonized expression for this operator which we have discussed gives

$$\begin{aligned} \langle\pi^+|Q_7^{(3/2)}|K^+\rangle \;&=\; \frac{3}{8\pi^2 f_\pi^2}\left[\sum_A f_A^2 M_A^6 \log\frac{M_A^2}{\Lambda^2} - \sum_V f_V^2 M_V^6 \log\frac{M_V^2}{\Lambda^2}\right] \\ &\simeq\; \frac{3}{4\pi^2}M_V^4\left(2\log 2 - \log\frac{\Lambda^2}{M_V^2}\right)\,, \end{aligned} \tag{35}$$

where in the second line we have used the LMD approximation discussed in ref. [20]. In this approximation and with the same numerical input as in (30), we obtain $\widetilde{B}_7^{(3/2)}(\mu = 2\,\mathrm{GeV}) \simeq +1.5$. However, as shown in Fig. 2, the value of this B factor is again very sensitive to the choice of the matching scale μ. The results obtained for the smaller value of the strange quark mass (*i.e.* for larger values of the condensate) are in agreement, within errors, with the numbers quoted in eq. (33). Unfortunately, the values of m_s (or of the condensate) and of the B factors obtained from the lattice are usually not quoted together.

In the case of Q_8, the large–N_c limit of eq. (24) simply reproduces the result obtained from the bosonized expression (17). Since subleading corrections in the $1/N_c$ expansion of this operator are not yet available, further comparison with lattice results for $B_8^{(3/2)}$ has to be postponed.

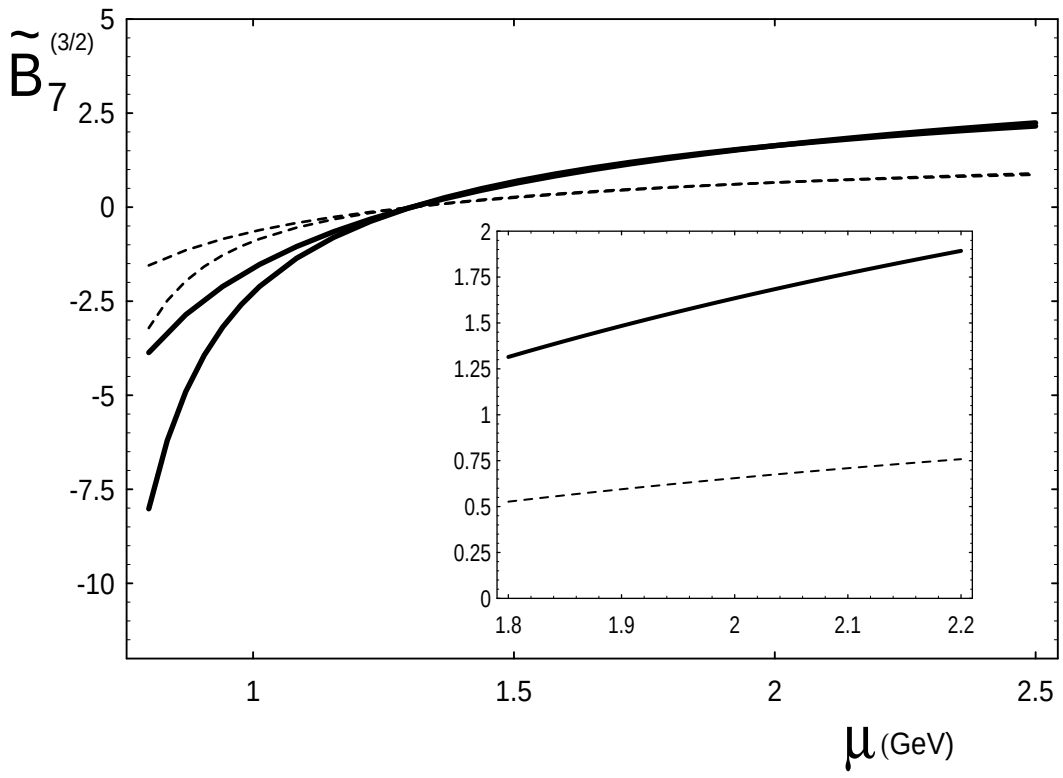

Fig. 2 *The B factor, $\tilde{B}_7^{(3/2)}$ [see eq. (31)], as a function of the μ scale in* GeV. *The solid lines correspond to $(m_s + m_d)(\mu = 2\,\mathrm{GeV}) = 158\,\mathrm{MeV}$; the dashed lines to the extreme low value $(m_s + m_d)(\mu = 2\,\mathrm{GeV}) = 100\,\mathrm{MeV}$.*

4 Conclusions and Outlook

The expressions in eqs. (22) and (35) are the main results reported in this letter. They are a first step towards a systematic evaluation of weak matrix elements in the chiral expansion and to first non–trivial order in the $1/N_c$ expansion. Our final aim, however, is to obtain values for the coupling constants of the low energy $\Delta S = 1$ and $\Delta S = 2$ chiral effective Lagrangian directly; i.e., constants like **h** in eq. (4) and not of individual matrix elements of four–quark operators. It is encouraging from the results obtained so far, to find such simple analytic expressions which exhibit only a logarithmic dependence on the matching scale μ; however, the fact that the numerical results are so sensitive to the choice of μ in the GeV region is perhaps an indication that one should be extremely cautious in the evaluation of errors of B factors, in general, both in model calculations and in lattice QCD numerical simulations.

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